Mechatronics Laboratory Session

Floating Ball Test

Name:

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1 Preface

The „floating ball“ laboratory test is part of the Mechatronics laboratory session for master students held in the summer term by the Institute of Dynamics and Vibration Research (IDS) together with the Institute of Automatic Control (IRT) and the Institute of Mechatronic Systems (imes). In this laboratory test, electromagnetic levitation is demonstrated with the help of a hollow ferromagnetic ball floating in a stable position. The test provides a basic knowledge of electronic circuits and their components and gives an introduction into the function principle of mechatronic systems.

To attend the lab successfully, it is necessary to read this script in advance. The script explains the physical modelling of the system, the controller structure and the behaviour of the closed-loop circuit. Thoroughly reading this script will provide you with all basic information needed for understanding the functioning of the electronic components and their circuit environment used in this laboratory test. Good luck!

The simple magnetic bearing which is a good example for a mechatronic system can be taken home after lab attendance. It requires a 12V 2A power supply and a globe-shaped pencil sharpener.

References

All previous knowledge necessary for dealing with this laboratory test is provided by this script. The literature listed at the end of the script is recommended to gather more detailed information on magnetic bearing technology. The book of Schweitzer [SCH94] is a good introduction into the topic. The book of Jung [JU88] offers basic knowledge of magnetic levitation technology. The book of Tietze/Schenck [TS93] belongs to the standard literature in the field of semiconductor circuitry. We can especially recommend the book of Horowitz [HO90] which is a lot more practice-oriented. Electronic component data sheets can also be found on the internet.

2 Introduction into Magnetic Bearing Technology

2.1 General

Magnetic bearings are typical examples for mechatronic systems. Additionally to the mechanical components, they also contain electronic components, as for example sensors, a controller – mostly a micro controller – as well as power electronics to control the actuators. With the increased significance of software, magnetic bearings can be used in multi-purpose and „intelligent“ fields of application. Magnetic bearings without control do not make any sense from the technical point of view. It is the combination of mechanics, sensors, actuators, control and data processing – called mechatronics – which gives rise to a functional machine element. Due to the progress made in semiconductor technology, high-performance and cost-efficient micro controllers and power semiconductors are available. Magnetic bearings thus become more and
more attractive to solve classical bearing problems in mechanical engineering. Nowadays, magnetic bearings are for example applied in vacuum technology, where lubricants of conventional bearings would evaporate and contaminate the vacuum. In machine tools, magnetically levitated milling spindles enable speeds of more than 30,000 min\(^{-1}\) required for high-speed cutting processes. In turbine construction, magnetic bearings guarantee a long lifetime even at high temperatures. Their contactless working principle and controllable dynamic design are further advantages of magnetic bearings. These properties permit for example new constructions, an operation without mechanical wear as well as high speeds and an active vibration damping.

In general, magnetic bearings utilise two different physical causes of magnetic effects:

1. The **reluctance force** acts perpendicular to the surface of materials of different permeability, aiming at minimum potential magnetic field energy. Reluctance is called the magnetic resistance, which is inversely proportional to the permeability \(\mu = \mu_r \mu_0\). Permeability depends on the relative permeability \(\mu_r\) which is a material property. Compared to air (\(\mu_r \approx 1\)), ferromagnetic materials have a very high relative permeability of \(300 \leq \mu_r \leq 10,000\).

2. The **Lorentz force** describes the electromagnetic force acting on charge carriers moving in the magnetic field. In technical applications, the movement of charge carriers corresponds to an electric current. According to equation

\[
\vec{F}_L = I \cdot \vec{\ell} \times \vec{B},
\]

the Lorentz force \(\vec{F}_L\) acts perpendicular to the technical direction of the current \(\vec{\ell}/\ell\) and to the magnetic flux density \(\vec{B}\), and it is proportional to the length \(\ell\) of the electrical conductor in the magnetic field, the electric current \(I\) and the magnetic flux density \(B\).

Based on these two possibilities for generating a force, Fig. 1 classifies 8 different types of magnetic bearings. Each bearing type including advantages, disadvantages, possible applications and so on is described in detail in [SCH94]. The floating ball experiment which is subject of this laboratory session belongs to bearing type 1, which is a „classical active magnetic bearing“.
Figure 1: Types of magnetic bearings according to [SCH94]
2.2 System Arrangement

The ball is intended to levitate in a stable position by means of a controlled electromagnet. This is only possible, when the reluctance force $F_m$ generated by the coil can counteract the weight $F_g$ acting on the ball. Since magnets just exert attractive forces on ferromagnetic bodies, the magnetic bearing is arranged as shown in Fig. 2.

![Diagram of magnetic bearing](image)

Equilibrium is reached, when the weight $F_g$ acting on the ball and the magnetic force $F_m$ cancel each other out. Due to the inhomogenous magnetic field of the coil, it is very difficult to calculate the reluctance force $F_m$. For this reason, the system behaviour shall be derived from the principally nonlinear curve of the reluctance force $F_m(X, I)$ in dependence of the coil current $I$ and the air gap $X$.

2.3 Behaviour of an Uncontrolled System

The force exerted by the electromagnet on the ball depends on the coil current $I$ and the air gap $X$. The force vs. current and force vs. distance curves of the electromagnet are shown in Fig. 3, linearised around the operating point $AP = (X_0, F_g)$.

![Graphs of magnetising curves](image)

The magnet’s force vs. distance curve shown in Fig. 3 is different to the one of a spring shown in Fig. 4. The curve of a linear spring has a positive slope. The deflection $X$ results in the spring force $F_i = cX$, which is opposed to the deflection $X$, trying to restore the deflection; thus
the spring force acts as a stabiliser. This is not valid for the magnetic force $F_m$. The magnetic force follows the inversely quadratic force/distance law $F_m \sim I^2/X^2$. The larger the distance $X$ between magnet and armature, the smaller the magnetic force. The sign of the slope of the force/distance function in the operating point decides on the stability. In magnetic bearing technology, this slope is also called the stiffness of a controlled system, analogously to the spring stiffness. A declining curve, as seen in Fig. 3, corresponds to negative stiffness. For this reason, uncontrolled electromagnets are instable and have to be stabilised by means of an appropriate controller.

In practice, dynamic system analysis and controller design are realised by using a linearised system model. For this purpose, the nonlinear curve $F_m(X,I)$ of the magnetic force is linearised in the operating point $I = I_0$, $X = X_0$, $F_m = F_g$. For controlling the „floating ball“, the deviations around the operating point (symbolised by small letters) are used as follows:

For the current this leads to

$$i = I - I_0 \ , \quad (2)$$

for the force to

$$f = F_m - F_g \ , \quad (3)$$

and for the distance to

$$x = X_0 - X \ . \quad (4)$$

With this, the total force

$$f(x,i) = - \frac{\partial F_m(X,I)}{\partial X} \bigg|_{AP} x + \frac{\partial F_m(X,I)}{\partial I} \bigg|_{AP} i = -k_x \cdot x + k_i \cdot i \quad (5)$$

can be given as a function of the deviations $x$ and $i$ linearised around the operating point $AP = (F_g, X_0, I_0)$. In this case, the deflection $x$ is counted positively, if the air gap $X$ gets smaller $^1$. This coordinate transformation changes the corresponding partial derivative to negative.

It must always be verified, if linearised differential equations are able to describe the dynamic system. Please note that Eq. (5) is suitable for controller design, because the curves do not show any jumps or ambiguities. For boundary cases, i.e. when ball and magnet get into contact or when the magnet is saturated, linearisation is not suitable any more.

$^1$In general, this is a matter of definition, which is not based on physics. In this case, the positive deflection $x$ corresponds to a resulting positive force $f$, which proves to be advantageous.
**Exercise:** Sketch the block diagram of an uncontrolled system according to Eq. (5) and determine the poles. Be aware of the direction of the deviation \( x \).

### 2.4 Controller and Closed-loop Circuit

A simple physical equivalent circuit model of a bearing consists of a damped spring/mass system. Here the body (mass \( m \)) to be supported is bound to its reference position due to the bearing stiffness \( c \). The damping constant \( b \) describes the structural damping of the bearing. The aim is to find a controller design, where the closed-loop circuit has the dynamic properties of a damped spring/mass system – called \( P_{T2} \) system in automatic control engineering.

![Block diagram](image)

\[
mx = -cx - bx
\]

\[
mx = k_x x + k_i i
\]

**Figure 5:** Comparison of physical equivalent circuit model and magnetic bearing

The controller is needed to limit the current \( i \) in such a way that the two bearings shown in Fig. 5 have comparable dynamic properties. Comparing the coefficients on the two right sides of the differential equations given in Fig. 5 leads to the new differential equation

\[
-cx - bx = k_x x + k_i i,
\]

which, solved for the control value \( i \), results in the controller equation

\[
i(x) = -\frac{(c + k_x)x + bx}{k_i}.
\]

Here \( c \) is the desired stiffness and \( b \) the desired damping of the magnetic bearing. The controller equation (7) keeps the magnetic bearing in a steady equilibrium position. Provided that the sensor acts as an ideal P element and that the power amplifier adjusts without delay the control value \( i \) given by the controller, both bearing models shown in Fig. 5 have the same dynamic properties.

Eq. (7) leads to the required controller structure (cf. Fig. 6):

\[
i(x) = -\frac{c + k_x}{k_i} x - \frac{b}{k_i} \dot{x} = K_p x + T_D \dot{x}.
\]

To create a technically practicable magnetic bearing, the deflection \( x \) at the bearing must be fed back via PD controller to the coil current.

**Exercise:** Why are P controllers not sufficient enough? Which are the additional advantages of PID controllers?

Compared to traditional bearings, magnetic bearings have the advantage that its dynamic properties can be adjusted:
Exercise: Draw a root locus (RL) of a PD controlled system according to Eq. (5)! For details how to sketch the RL, please refer to [IRT].
3 Controller Designed as Operational Amplifier Circuit

To stabilise the system, the controller is designed by means of operational amplifiers (op amps). This chapter describes the behaviour and some basic circuits of operational amplifiers. The operational amplifier is considered as an ideal amplifier, and effects like drift, noise, rise time etc. are neglected.

3.1 Open-loop Operational Amplifier

An operational amplifier (cf. Fig. 7) amplifies the voltage difference \( U_D = U_P - U_N \) by the gain \( A_0 \):

\[
A_0 = \frac{U_A}{U_D} = \frac{U_A}{U_P - U_N}.
\]

The gain is very high and is about \( A_0 = 100000 \) [NS89] for the operational amplifier LM324 used here. Fig. 8 shows that the output voltage \( U_A \) is limited to a negative \( U_{A,\text{min}} \) or positive \( U_{A,\text{max}} \) depending on the operating voltage. The curve is linear between \( U_{A,\text{min}} \) and \( U_{A,\text{max}} \); the operational amplifier has a proportional gain of \( A_0 = \Delta U_A / \Delta U_D \).

In practical use, operational amplifiers are often considered as ideal amplifiers with nearly infinitely high gain and infinitely high input resistance and a negligible output resistance.
3.2 Feedback Operational Amplifier

Due to the high gain of operational amplifiers, a low voltage $U_D$ already leads to a saturation of the output voltage $U_A$. In addition, the so-called no-load gain $A_0$ of an operational amplifier has a high manufacturing tolerance – in practice several hundred percents – and is largely temperature-dependent. For this reason, operational amplifiers are designed in many applications with a feedback loop, in order to feed back the output voltage to the negative input of the operational amplifier (cf. Fig. 9). Thanks to the block diagram algebra, the feedback loop can be eliminated, and the resulting gain

$$A_G = \frac{U_A}{U_E} = \frac{A_0}{1 + F_R(s) A_0}$$

of the feedback operational amplifier can be derived. Due to the high gain $A_0$ of the operational amplifier, $|F_R(s) A_0| \gg 1$ and Eq. (10) can be simplified to

$$A_G = \frac{U_A}{U_E} = \frac{1}{F_R(s)}.$$  (11)

This means that the gain of the circuit is determined by the feedback network alone. Compared to the deviations in semiconductor components, the tolerances of passive components used in the feedback network are in practice on a very low level.

![Figure 9: Basic principle of feedback](image)

Using an operational amplifier, a variety of mathematical operations like addition, subtraction, differentiation and integration can be realised. Due to their high accuracy, operational amplifiers are not only suitable for simulating mathematical operations in analog computers, but can also be used in analog controls.

3.3 Basic Op Amp Circuits

3.3.1 Non-inverting Amplifier

A variable feedback factor can be integrated into the circuit by using a simple voltage divider with the factor $k = R_1/(R_1 + R_0)$. Fig. 10 shows a non-inverting amplifier circuit. The gain

$$A_G = \frac{U_A}{U_E} = \frac{1}{\frac{R_1}{R_1 + R_0}} = 1 + \frac{R_0}{R_1}$$

is always positive, i.e. $A_G > 1$. 

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3.3.2 Inverting Amplifier

Fig. 11 shows the circuit diagram of an inverting amplifier. Since the gain $A_0 = \frac{U_A}{U_D}$ is very high, the voltage difference $U_D$ between the two inputs of the operational amplifier tends to be very small.

For the circuit shown in Fig. 11, the positive input of the operational amplifier is at earth potential. Due to its high gain, the operational amplifier „forces“ the negative input to be at earth potential as well. This effect is known as „virtual earth“. As the current flowing at the operational amplifier’s input is negligibly small, the gain of the inverting amplifier can be determined by applying Kirchhoff’s first law

$$\sum i = 0 = \frac{U_A}{R_0} + \frac{U_E}{R_1}. \quad (13)$$

The difference between output and input voltage can be expressed by the relation:

$$A_G = \frac{U_A}{U_E} = -\frac{R_0}{R_1}. \quad (14)$$

3.3.3 Summing Amplifier

A summing amplifier is an inverting amplifier, where several input voltages $U_1, \ldots, U_n$ are applied via the resistances $R_1, \ldots, R_n$ to its negative input (cf. Fig. 12).

Due to the high gain $A_0$, the current flowing into the amplifier at node S is neglected here too. From Kirchhoff’s first law

$$\frac{U_1}{R_1} + \frac{U_2}{R_2} + \ldots + \frac{U_n}{R_n} + \frac{U_A}{R_0} = 0, \quad (15)$$
the output voltage results in
\[ U_A = -R_0 \left( \frac{U_1}{R_1} + \frac{U_2}{R_2} + \ldots + \frac{U_n}{R_n} \right) = a_1 U_1 + a_2 U_2 + \ldots + a_n U_n \] (16)
as superposition of the input voltages \( U_i \) scaled with the constant factor \( a_i \). The scaling can be varied via the ratio \( a_i = -R_i/R_0 \).

### 3.3.4 Differentiator

Instead of frequency-independent components used in the feedback network, frequency-dependent components – mostly capacitors, because precise inductors are more difficult to produce – can be used. This leads to frequency-dependent feedback, specifically influencing the amplifier’s frequency response. This approach is also used for active filters.

A capacitor’s current
\[ i_C = C \frac{d u_C}{dt} \] (17)
is proportional to the rate of change of the capacitor voltage \( u_C \). The coefficient of proportionality is the capacitance \( C \), measured in farad \( \text{F} = 1 \frac{\text{As}}{\text{V}} \). In Fig. 13, the resistor \( R_1 \) from Fig. 11 is replaced by a capacitor. Kirchhoff’s first law leads to the output voltage
\[ U_A = -R C \frac{d U_E}{dt} \] (18)
as time derivative of the input voltage \( U_E \); the circuit shown in Fig. 13 is consequently a differentiator. For a sinusoidal input voltage
\[ U_E(t) = \hat{U}_E \sin(\omega t), \] (19)
the differentiator provides a sinusoidal output voltage

\[ U_A(t) = -\hat{U}_E RC \omega \cos(\omega t) \]  (20)

of the same frequency. The differentiator’s amplitude gain \( A \)

\[ A = \frac{\dot{U}_A}{\dot{U}_E} = \frac{RC \omega \hat{U}_E}{\hat{U}_E} = \omega RC \]  (21)

increases with rising frequency and causes an undesired amplification of high-frequency noise in the input signal. In practice, a resistor is therefore always switched in series to the capacitor, producing a \( \text{D}_{\text{T}} \) behaviour instead of an ideal D behaviour.

### 3.3.5 Integrator

If replacing in the circuit of Fig. 11 the resistor \( R_0 \) by a capacitor, the resulting amplifier gets an integrating behaviour.

![Integrator](image)

Figure 14: Integrator

An integrator supplies the output voltage \( U_A \), which is proportional to the integral of the input voltage \( U_E \). Fig. 14 shows how an integrator can be realised by means of an operational amplifier. From Kirchhoff’s first law

\[ \frac{U_E}{R} + C \frac{dU_A}{dt} = 0, \]  (22)

the output voltage

\[ U_A = U_A(t_0) - \frac{1}{RC} \int_{t_0}^{t} U_E \, dt \]  (23)

is derived as the integral of the input voltage \( U_E \) with respect to time. A sinusoidal input voltage

\[ U_E(t) = \dot{U}_E \sin(\omega t) \]  (24)

leads to a sinusoidal output voltage

\[ U_A(t) = \dot{U}_E \frac{1}{\omega RC} \cos(\omega t) \]  (25)

of the same frequency. The integrator’s amplitude gain

\[ A = \frac{\dot{U}_A}{\dot{U}_E} = \frac{\dot{U}_E}{\omega RC \dot{U}_E} = \frac{1}{\omega RC} \]  (26)

quickly decreases for higher frequencies. One problem of the integrator is its initial value \( U_A(t_0) \). In analog computers, the initial value of integrators is for example adjusted by presetting
4 Components

The „floating ball“ circuit contains many components which are also used in analog controls. This chapter gives a short introduction into the functioning of these components. More detailed information can be found in [HO90] or on various web pages, e. g. [ELKO].

4.1 Quadruple Operational Amplifier

In practice, the operational amplifier described in Section 3.1 is mostly designed as a quad operational amplifier. This is a 14-pin DIL\(^2\) IC, incorporating 4 operational amplifiers due to space restrictions. The 4 op amps are all powered via pin 4 and 11 (cf. Fig. 15). The identifying notch on the package identifies the position of pin 1 and pin 14.

![Figure 15: Op amp pin numbering](image)

4.2 Voltage Regulator

The voltage regulator regulates a given input voltage down to a desired lower output voltage. Type 7805 used here is a 5V linear voltage regulator (voltage drop via regulated resistor). The waste heat is dissipated by a heatsink. Type 7805 is incorporated together with the power transistor described below in a TO-220 housing with pin numbering and simplified circuit diagram as indicated in Fig. 16. For more functioning details, refer to [TS93]. A safe operation additionally requires the use of capacitors at the voltage regulator’s in- and outputs, in order to avoid a vibration of the control.

\(^2\)Dual in-line package with a housing and two parallel rows of electrical connecting pins.
4.3 Power Transistor

A type BD645 power transistor is used here; its pin numbering and principle structure is shown in Fig. 17. These are two transistors in cascade connection. Transistors are 3-pin semiconductors, where a smaller base current regulates a much larger collector current. [ELKO] gives a good introduction into this topic. In the „floating ball“ test, the transistor is used as an amplifier, in order to convert the weak output signal of the current controller into an appropriately large coil current. One single power transistor alone has a relatively low current gain of about 5 to 20. If the power transistor’s base is activated by a so-called small-signal transistor with a high current gain of about 300 to 500, the two current gains are multiplied. This variant is called a Darlington transistor just requiring small controlling currents (less power needed) to control high output currents.

4.4 Diodes and Zener Diodes

Diodes are semiconductor electronic devices, allowing an electric current to pass in one direction only. The diode conducts the current from plus to minus, i. e. from anode to cathode. The cathode (minus) is depicted by a thin bar on the diode package. So diodes can be used to convert alternating currents into direct currents. In this circuit, they are used as protection against polarity reversal. Zener diodes are special diodes allowing an electric current also to flow in the reverse direction. The diode is reverse-biased above its reverse breakdown voltage, having a very low differential resistance. In this region, the voltage drop at the diode is nearly independent of the current flow. In forward direction, the Zener diode functions in the same manner as a normal diode.

4.5 Other Components

For larger components, e. g. potentiometers and electrolytic capacitors, the values are directly imprinted on the component. For smaller components, a code is used:

- Resistors are color coded in accordance with DIN IEC 62 (see Annex). In this laboratory test, resistors with 4 bands (carbon film resistors) or with 5 bands (metal film resistors) are used. The color code is composed of 2 or 3 bands for the numerical value, one band for the multiplicator and one band for the tolerance. The resistor values are graded according
to E series for resistors (E3 to E96): resistors with smaller tolerances belong to higher series with finer grading.

- Smaller capacitors mostly have a three-digit number. Similar to the resistors, the first two digits give the component’s value multiplied with the 3rd digit as a power of ten with the unit pF (10^{-12} F). Example: 103 = 10 \cdot 10^3 \text{ pF}, i. e. 10 \text{ nF}.

5 Hardware Components

The hardware components are described based on the circuit diagram given in the Annex, Fig. 18.

5.1 Power Supply

The “floating ball”s“ power supply is realised by means of a plug-in adapter where the DC voltage is supplied to the PCB’s plus and minus terminals (jumpers J1-1 and J1-2). The diode D1 is used as protection against polarity reversal. The capacitor C3 is a reservoir capacitor smoothing the supply voltage.

Operational amplifiers normally require a positive and a negative supply voltage. For this reason, the negative supply voltage is supplied by the voltage regulator Q2 (7805), which regulates the output voltage between pin 2 and 3 to 5V, independant of the input voltage. The capacitor C4 suppresses possible vibrations of the voltage regulator. The 5 V output voltage supplies the lamp of the sensor, and a series resistor R1 adjusts the 5 V to the lamp voltage of about 3 V.

The output of the voltage regulator is used as ground of the operational amplifier. Due to this increase in reference potential, no separate negative power supply of the operational amplifier is needed; ground (0 V), -5 V and +7 V are thus available.

5.2 Sensor

The sensor measuring the air gap between ball and magnet consists of a lamp and a large-scale photoelectric cell (solar cell). The sensor’s measuring principle is based on the fact that the solar cell is shadowed by the ball: The smaller the air gap between ball and magnet, the smaller the quantity of light reaching the solar cell. The current of the solar cell is proportional to the light quantity hitting the solar cell and is thus nearly proportional to the air gap, i. e. to the position of the floating ball.

The photoelectric current $i_{\text{Sensor}}$ is converted by the inverting operational amplifier U2C into a proportional voltage $u_{\text{Sensor}} = i_{\text{Sensor}} R_2$. Together with the resistor R2, the capacitor C1 acts as a low pass filter with a corner frequency of approx. 15 kHz, suppressing noise and disturbances of the sensor (e. g. caused by a medium-wave transmitter).

The inverting input of the operational amplifier is connected to the cathode of the solar cell. So the two negative signs (negative current direction and gain) cancel each other out.
5.3 Setpoint Setting

The setpoint which determines as reference variable the air gap between ball and magnet is applied by the potentiometer R12 and the subsequent voltage follower U2B to the P controller. The voltage follower is realised by a non-inverting amplifier (cf. Fig. 10) with $R_0 = 0$ and $R_1 \to \infty$; according to Eq. (12), it has a gain of 1 and is characterised by a high input resistance for a small output resistance. The output voltage of U2B can be set by the potentiometer R12 in the range of 0 V to -2.5 V. The subsequent P controller is switched as summing amplifier, providing proportional feedback $K_P(y - w)$. The optional switch SW1 can be used to initiate a setpoint change.

5.4 Current Control

The summing amplifier composed of U2A and R6, R7 and R8 regulates the current controller U2D. The summing amplifier’s reference potential is 0 V; the one of the subsequent current control however is with -5 V equal to the initial ground at pin 2 of the voltage controller Q. These two potentials are adjusted by means of the Zener diode D2 with a voltage drop of approx. 5.6 V. The resistor R9 guarantees that Zener current constantly flows through the Zener diode. The output of U2A is thus about 5.6 V higher than the non-inverting input of U2D.

As shown in Fig. 18, U2D together with the Darlington transistor Q1 forms a power current amplifier. The output of U2D controls the base of Q1 which supplies the electromagnet as Darlington emitter follower. The current flowing through the electromagnet is measured by means of R10 and is applied to the inverting input of U2D. If the voltage at the non-inverting input of U2D is higher than the one at the inverting input, i.e. the reference variable is larger than the actual value, a positive voltage is applied to the output of U2D, increasing the Darlington transistor’s gain and thus the coil current. If the reference variable is smaller than the actual value, the output of U2D goes to low saturation $U_{A_{\text{min}}}$, just slightly above 0 V. This results in a low gain of the Darlington transistor, i.e. the coil current is reduced.

5.5 Controller Design

The controller compares the measured variable $x$ and the reference variable $-w$. The system deviation $x - w$ is fed via the P controller U1B and the controlled variable $x$ is additionally fed via the D controller U1A to the two inputs of the summing amplifier U2A. This results in the parallel structure, as developed in Fig. 6. The negative feedbacks $K_P < 0$ and $T_D < 0$ are realised according to Eq. (8) by means of inverting operational amplifiers based on Eq. (14) and Eq. (18). The negative sign of the summing amplifier is compensated by a change of sign in the sensor $^4$.

- The P controller is realised with U1 B. This requires a summing amplifier, which adds the

$^4$In contrast to Section 2.3, the deviation $x$ is directed downwards; a larger air gap results in a larger photoelectric current of the solar cell, thus leading to a voltage increase at the output of U2C.
controlled variable $y$ and the setpoint $-w$, then multiplying it with the gain $K_p$. It must be possible to adjust the gain $K_p$ between 0 ...-50 by means of the potentiometer R13 (0-500 kΩ).

**Exercise:** Calculate the resistances for R4 and R5!

- The $D_{T1}$ controller shall be realised according to Fig. 13. Replacing the capacitor by a capacitor and resistor connected in series leads to the desired $D_{T1}$ behaviour, because for high frequencies the gain is only determined by the R14 / R3 ratio. If R14 is set to 500 kΩ, the gain shall amount to -15 for 10 Hz. At this frequency, the resistance of R3 can be neglected with respect to the impedance of the capacitor C2.

**Exercise:** Calculate the value for C2!

C2 creates together with R3 a RC combination with a limiting frequency of $f_{gr} = \frac{1}{\pi R_C} = 300$ Hz.

**Exercise:** Calculate the value for R3!

### 6 Installation and Commissioning

#### 6.1 Controlled System

The electromagnet consists of a M8 x 80 screw, 2 washers and one nut. Enamelled copper of 0.42 mm diameter is wound between the washers with a distance of approx. 50 mm. About 700 turns are wound in the winding space, so that the coil’s outer diameter is approx. 15 mm.

The wooden frame of the „floating ball“ experiment comprises a base plate, two side parts and a cross beam, to which the electromagnet is fixed. The light bulb is screwed with its fitting into the left side part about 1 cm below the bottom end of the magnet. The solar cell must be fixed to the right side part, so that the electromagnet’s shadow barely hits the upper edge of the solar cell.

Electromagnet, lamp and solar cell are plug-connected to the printed circuit board (PCB). Be aware of the solar cell’s correct polarity, since polarity reversal would lead to an instable closed-loop circuit.

#### 6.2 PCB Assembly

The PCB is populated with all necessary components. Following the assembly plan on the PCB’s upper side, the small components like resistors should be assembled first. It is essential to pay attention to the correct polarity of the electrolytic capacitors C3 and C4, of the ICs U1 and U2 as well as of the diodes D1 and Z1. A quadratic solder land identifies the plus pole (C3, C4), pin 1 (U1, U2) and the cathode depicted by a thin bar (D1, Z1).

There are two alternatives for small- and large-size assembly of the potentiometers R12-R14. The jumpers J2-J6 may have solder pins to facilitate later measurements. J7 and J8 remain
empty.
The power semiconductors Q1 and Q2 are cooled by a cooling plate. The two semiconducters must be isolated against each other: A mica washer is placed between connecting lug and cooling plate, and the power semiconductor is fixed by a screw with insulating sleeve to the cooling body and the PCB.

6.3 Sensor Testing

Switch on power supply to test the sensor, and measure the voltage at terminal Y. It must be positive (approx. 1 V, depending on the solar cell). The voltage decreases, when the solar cell is shadowed. If this does not happen, there is polarity reversal.

6.4 PD Controller Tuning

The PCB is assembled with three potentiometers, with which the two controller parameters and the setpoint can be adjusted.

1. The P component $K_p = -\frac{(c + k_x)}{k_i}$ specifies the desired stiffness of the system, because $k_x$ and $k_i$ as coefficients of the linearised differential equation have fixed values.

2. The desired damping can be adjusted by using the $T_D$ potentiometer. Without damping, the floating ball is theoretically an undamped, vibratory system. Caused by the inertia of the sensor and especially of the actuator (coil inductance), the stability limit is reached much earlier. For this reason, the floating ball requires an appropriate damping.

3. The desired ball position $x_{soll}$ can be adjusted in a wide range by means of the potentiometer on the right hand side. Please note that a large distance between magnet and ball requires a correspondingly large coil current. Current limiting of the power supply unit causes the ball to drop.

When starting, the potentiometers are turned for $T_D$ and $K_p$ to their mid-position. Then the ball is manually held – without shadowing the sensor beam! – approx. 10 mm below the electromagnet, and the setpoint is fed back until the electromagnet attracts the ball. In case of a sudden attraction of the ball, the P component must be reduced.

This laboratory test is quite cost-intensive from the components’ side. For this reason, the hollow steel ball and the plug-in adapter must be bought by the participants themselves:

- One globe-shaped pencil sharpener (available at stationery shops, approx. 1 Euro)
- One plug-in adapter 12V/800 mA DC, unstable (available at electronic supply shops, approx. 10 Euro). For later experiments, a larger power supply unit is required (12V/1.5A).

Note: The circuit must not be operated without the ball over a longer period of time. Without the ball, the Darlington transistor is set to maximum gain with a current of several amperes, which would overload the power semiconductors and the power supply unit.
References


Figure 18: Circuit diagram
### Part list

<table>
<thead>
<tr>
<th>Part</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>C1</td>
<td>2.2nF Ceramics capacitor</td>
</tr>
<tr>
<td>C2</td>
<td>to be calculated Foil capacitor</td>
</tr>
<tr>
<td>C3</td>
<td>1000µF/16V Electrolytic capacitor</td>
</tr>
<tr>
<td>C4</td>
<td>47µF/16V Electrolytic capacitor</td>
</tr>
<tr>
<td>D1</td>
<td>1N4001 Universal diode</td>
</tr>
<tr>
<td>J1</td>
<td>8-pin strip for connections</td>
</tr>
<tr>
<td>J2-6</td>
<td>Solder pins as measuring points</td>
</tr>
<tr>
<td>J7-8</td>
<td>Solder pins optional</td>
</tr>
<tr>
<td>Q1</td>
<td>BD645 Darlington transistor</td>
</tr>
<tr>
<td>Q2</td>
<td>7805 Voltage regulator</td>
</tr>
<tr>
<td>R1</td>
<td>10Ω/0.5W Resistor</td>
</tr>
<tr>
<td>R2</td>
<td>4.7kΩ Resistor</td>
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<tr>
<td>R3-5</td>
<td>to be calculated Resistor</td>
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<tr>
<td>R6-9</td>
<td>10kΩ Resistor</td>
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<tr>
<td>R10</td>
<td>0.47Ω/1W Resistor</td>
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<tr>
<td>R11</td>
<td>470kΩ Resistor</td>
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<tr>
<td>R12-14</td>
<td>500kΩ Potentiometer</td>
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<tr>
<td>R15</td>
<td>150kΩ Resistor</td>
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<tr>
<td>SW1</td>
<td>Switch optional</td>
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<tr>
<td>U1-2</td>
<td>LM324 Quadruple operational amplifier</td>
</tr>
<tr>
<td>Z1</td>
<td>BZX79C5V6 Zener diode 0.3W</td>
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### Installation

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<td>1</td>
<td>Wooden frame</td>
</tr>
<tr>
<td>1</td>
<td>PCB</td>
</tr>
<tr>
<td>1</td>
<td>Screw M8x80 with nut and 2 washers</td>
</tr>
<tr>
<td>100g</td>
<td>CuL wire 0.42mm</td>
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<tr>
<td>1</td>
<td>Solar cell approx. 25mmx10mm</td>
</tr>
<tr>
<td>1</td>
<td>Lens light bulb 3.7V/0.3A with socket</td>
</tr>
<tr>
<td>1</td>
<td>Alu sheet 100mmx70mm as cooling body</td>
</tr>
<tr>
<td>2</td>
<td>Insulating sleeves</td>
</tr>
<tr>
<td>2</td>
<td>Insulating washers</td>
</tr>
<tr>
<td>2</td>
<td>Screws M3x10 with nut</td>
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</tbody>
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### Component Colour Codes

#### Four Band Resistors

<table>
<thead>
<tr>
<th>1st band</th>
<th>2nd band</th>
<th>3rd band</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brown 1</td>
<td>Brown 7</td>
<td>Black 3</td>
<td>±1%</td>
</tr>
<tr>
<td>Brown 2</td>
<td>Brown 8</td>
<td>Black 4</td>
<td>±2%</td>
</tr>
<tr>
<td>Orange 3</td>
<td>Orange 9</td>
<td>Black 5</td>
<td>±5%</td>
</tr>
<tr>
<td>Orange 4</td>
<td>Orange 10</td>
<td>Black 6</td>
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</tr>
<tr>
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<td>Violet 11</td>
<td>Black 7</td>
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<td>Violet 6</td>
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#### Five Band Resistors

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<th>1st band</th>
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<th>4th band</th>
<th>Tolerance</th>
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<tbody>
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<td>Brown 3</td>
<td>Brown 4</td>
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<tr>
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<td>Orange 4</td>
<td>Orange 5</td>
<td>±5%</td>
</tr>
<tr>
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<td>Orange 10</td>
<td>Orange 6</td>
<td>Orange 7</td>
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<tr>
<td>Yellow 5</td>
<td>Yellow 11</td>
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<td>Yellow 7</td>
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<tr>
<td>Yellow 6</td>
<td>Yellow 12</td>
<td>Yellow 8</td>
<td>Yellow 9</td>
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#### Six Band Resistors

<table>
<thead>
<tr>
<th>1st band</th>
<th>2nd band</th>
<th>3rd band</th>
<th>4th band</th>
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<th>Temp Coef (PPM/C)</th>
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<tbody>
<tr>
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<td>Brown 4</td>
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<td>±1%</td>
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<tr>
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<td>Orange 5</td>
<td>Orange 6</td>
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<tr>
<td>Orange 4</td>
<td>Orange 10</td>
<td>Orange 6</td>
<td>Orange 7</td>
<td>Orange 8</td>
<td>±10%</td>
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<tr>
<td>Yellow 5</td>
<td>Yellow 11</td>
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<td>Yellow 7</td>
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<td>Yellow 10</td>
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